Int J Theor Phys (2009) 48: 684–688
DOI 10.1007/s10773-008-9843-6

Nonlocality and the Correlation of Measurement Bases

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Received: 26 June 2008 / Accepted: 26 August 2008 / Published online: 10 September 2008

Abstract Nonlocal nature apparently shown in entanglement is one of the most striking features of quantum theory. We examine the locality assumption in Bell-type proofs for entangled qubits, i.e. the outcome of a qubit at one end is independent of the basis choice at the other end. It has recently been claimed that in order to properly incorporate the phenomenon of self-observation, the Heisenberg picture with time going backwards provides a consistent description. We show that, if this claim holds true, the assumption in nonlocality proofs that basis choices at two ends are independent of each other may no longer be true, and may pose a threat to the validity of Bell-type proofs.

Keywords Nonlocality · Basis choices · Entanglement

The nonlocal nature exhibited in quantum entanglement is arguably the most distinctive departure from classical physics. After Bell's pioneering work [1] in testing nonlocality in entangled quantum systems, a number of variations of theoretical models [2–5] and experimental confirmations have followed [6]. While these truly marvelous results appear to have confirmed the validity of quantum theory and triumphed over locality imposed by relativity, subtle related issues remain. That is, although quantum theory indeed appears to possess nonlocality, this property cannot be used for superluminal signalling. This is rather puzzling because there seems to exist faster-than-light influencing yet superluminal signalling is not allowed. Another puzzling feature related to entanglement is the negativity shown in the conditional entropy of entangled quantum systems [7]. While a number of interpretations have been made with regard to this negativity [8], this issue is still considered to be unsettled [9]. Due to this negativity, Cerf and Adami [7] have proposed to interpret entanglement as a qubit and anti-qubit correlation where anti-qubit is a qubit traveling backwards in time.

In Bell-type inequality proofs such as that of Clauser-Horne-Shimony-Holt [2], one of the critical assumptions is the locality condition: that is, the outcome of a particle, say at Alice's end, is independent of the basis choice at the other end, or at Bob's end. This then

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implies that the basis choice at one end cannot be correlated with the basis choice at the other end. This is because if the basis choice were correlated, i.e., if the basis choice at one end is influenced by the basis choice at the other end, then a qubit at Bob's end could learn about Alice's basis choice through the measurement basis at Bob's end. The goal of this paper is to examine this particular assumption made in Bell-type proofs, i.e., we wish to consider if the assumption that basis choices at two ends are uncorrelated is sound and solid. In order to examine this assumption in fuller detail, let us review the concept of the physical reality defined by Einstein, Podolsky, and Rosen (EPR) [10] as follows:

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*

Let us consider EPR's definition of physical reality with a single qubit. Let us denote the value of a physical quantity for a given qubit as N_Q. A measurement on the single qubit can be done with a certain choice of observable and the outcome would be ± 1. If we denote the choice of observable as N_C, then the value of a physical quantity, i.e., the physical reality for the qubit, can be written as follows,

$$N_Q(N_C) = \pm 1. \tag{1}$$

That is, the physical reality of the qubit is obtained as a function of the choice of measurement basis. We now wish to examine the physical reality of the choice of measurement basis, i.e., N_C. Certainly, there are a number of ambiguous elements if we wish to discuss the physical reality of the basis choices. For instance, it is unclear what makes the decision to choose certain basis, i.e., is it the computer random number generator, a detector, or a human brain. Instead of considering the physical reality of basis choice as a function of these ambiguous elements, we wish to consider the N_C as a function of the value of a physical quantity, i.e., N_Q. In doing so, there is a big assumption since the decision of making an observable choice takes place prior to the eigenvalue outcome, we are considering the physical reality of the past event as a function of the future event.

In [11, 12], the observables were considered as observer's reference frame in observing and measuring a given quantum state. In particular, it was argued that when it is the unitary transformation of the observer's reference frame that is observed by the same observer, both the Schrödinger and the Heisenberg pictures cannot describe this phenomenon consistently. It was then argued [11] that in order to correctly describe the observation of observer's own reference frame is to consider the Heisenberg picture with time going backwards, i.e., it is the observables, or the observer's reference frame, that is evolving backwards in time. With this argument in [11], let us consider a simple example. Suppose for a given qubit $|0\rangle$, an observer is to measure this qubit either in Z- or X-basis. If the observer chooses Z-basis, then he applies an identity operator and measure with $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$. When X-basis is chosen, the observer applies a Hadamard gate, $H \equiv \frac{1}{2}(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|)$, to the given qubit and measures σ_z and the eigenvalue outcome ± 1 would be obtained. In the Heisenberg picture, rather than the state being unitarily transformed, it is the basis choice that is evolving. For convenience, suppose at $t = 0$, the observer chooses to measure either in Z- or X-basis. Then either $\mathbf{1}$ or H is applied to Z basis and Z or $HZH = X$ is obtained. Since we are taking an assumption that the unitary transformation is being done in time backward manner, when the eigenvalue outcome is obtained at $t = -1$,[1] and the eigenvalue

[1] We take the time difference in integer values for convenience.

of ± 1 would be obtained. Therefore, we see that if we take the result in [11], the eigenvalue outcome (at $t=-1$) takes place prior to the basis choice at $t=0$. This justifies our method of considering the physical reality of N_Q as a function of N_C if we assume the claim in [11] is correct.

Let us suppose that two spatially distant parties, called Alice and Bob, share maximally entangled qubits Q_1 and Q_2, as follows:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_{Q_1}|0\rangle_{Q_2} - |1\rangle_{Q_1}|1\rangle_{Q_2}). \tag{2}$$

Now suppose Alice and Bob choose to measure either in Z or X basis with $P=1/2$, respectively, and the eigenvalue outcomes ± 1 are obtained at each end. Let us focus our attention to two events taking place at two different times, i.e., when the basis choice is made which we will set to be at $t=0$ and when the eigenvalue outcome is obtained at $t=-1$. That is, the eigenvalue outcome is a prior event than the basis choice and we wish to examine correlation of basis choices with respect to the eigenvalue outcomes. Deutsch and Hayden gave a useful notation in describing the entangled qubits in the Heisenberg picture. However, for our purpose, it would be more convenient to discuss the correlation of basis choices with respect to the eigenvalue outcomes in the Schrödinger picture. Although in [11], it was shown that the Heisenberg picture that gives a correct picture rather than the Schrödinger picture, this only applies to a very special case, just as Newton's theory is much more convenient and sufficient method in most cases even though relativity is the correct description. Since it would more convenient in analyzing the basis choices with respect to eigenvalue, we wish to write down the process in the Schrödinger picture. The only thing to remember is that we will consider the basis choice with respect to the eigenvalue outcomes.

We will assume Alice and Bob each possess extra qubits Q_3 and Q_4, which will be used to represent their choice of measurement basis, respectively. Therefore, Alice has Q_1 and Q_3, while Bob contains Q_2 and Q_4 at his end. Now, Alice and Bob can measure Q_1 and Q_2 in either Z or X bases with $P=1/2$, respectively. If Alice chooses to measure Q_1 in Z basis, she prepares Q_3 to be $|0\rangle_{Q_3}$ and does not apply any unitary operation on Q_1. If Alice wants to measure in X basis, she then prepares $|1\rangle_{Q_3}$ and applies the Hadamard operation, H, to Q_1. Therefore, the outcome of Q_3 will indicate the chosen basis while the outcome of Q_1 will be either $+1$ or -1. It should be noted that we will consider Q_3 and Q_4 represent observers' choice of observables. Following the suggestion in [11], we will consider Q_3 and Q_4 as functions of Q_1 and Q_2.

Alice could choose Z or X by preparing Q_3 to be in $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{Q_3}$ initially, then projecting it onto $\{|0\rangle, |1\rangle\}$. If the outcome is $+1$, then she doesn't apply any unitary operation on Q_1, while Hadamard gate is applied for -1 outcome of Q_3. Or Alice can toss a coin[2] and prepare $|0\rangle_{Q_3}$ for heads while preparing $|1\rangle_{Q_3}$ and applying Hadamard to Q_1 for tails. Likewise, Bob can prepare $|0\rangle_{Q_4}$ if he wishes to measure in Z while preparing $|1\rangle_{Q_4}$, and apply H to Q_2 when it is to be measured in X basis. The final state for qubits Q_1, Q_2, Q_3, and Q_4 can then be written as follows:

$$\rho = \frac{1}{4}|0\rangle_{Q_1}\langle 0| \otimes |0\rangle_{Q_2}\langle 0| \otimes \Big[\frac{1}{2}|0\rangle_{Q_3}\langle 0| \otimes |0\rangle_{Q_4}\langle 0| + \frac{1}{4}|0\rangle_{Q_3}\langle 0| \otimes |1\rangle_{Q_4}\langle 1| + \frac{1}{4}|1\rangle_{Q_3}\langle 1| \otimes |0\rangle_{Q_4}\langle 0|\Big]$$

[2] We assume the coin used by Alice had no previous contact with the coin used at Bob's end.

$$
\begin{aligned}
&+\frac{1}{4}|0\rangle_{Q_1}\langle 0|\otimes|1\rangle_{Q_2}\langle 1|\otimes\Big[\frac{1}{2}|1\rangle_{Q_3}\langle 1|\otimes|1\rangle_{Q_4}\langle 1|\\
&+\frac{1}{4}|0\rangle_{Q_3}\langle 0|\otimes|1\rangle_{Q_4}\langle 1|+\frac{1}{4}|1\rangle_{Q_3}\langle 1|\otimes|0\rangle_{Q_4}\langle 0|\Big]\\
&+\frac{1}{4}|1\rangle_{Q_1}\langle 1|\otimes|0\rangle_{Q_2}\langle 0|\otimes\Big[\frac{1}{2}|1\rangle_{Q_3}\langle 1|\otimes|1\rangle_{Q_4}\langle 1|\\
&+\frac{1}{4}|0\rangle_{Q_3}\langle 0|\otimes|1\rangle_{Q_4}\langle 1|+\frac{1}{4}|1\rangle_{Q_3}\langle 1|\otimes|0\rangle_{Q_4}\langle 0|\Big]\\
&+\frac{1}{4}|1\rangle_{Q_1}\langle 1|\otimes|1\rangle_{Q_2}\langle 1|\otimes\Big[\frac{1}{2}|0\rangle_{Q_3}\langle 0|\otimes|0\rangle_{Q_4}\langle 0|\\
&+\frac{1}{4}|0\rangle_{Q_3}\langle 0|\otimes|1\rangle_{Q_4}\langle 1|+\frac{1}{4}|1\rangle_{Q_3}\langle 1|\otimes|0\rangle_{Q_4}\langle 0|\Big]\\
&+\text{off-diagonal terms.}
\end{aligned}
\tag{3}
$$

We now discuss the state ρ in (3) in terms of elements of reality. By following the logic structure used in Hardy's proof [5], with the state ρ in (3), we would like to consider elements of physical reality for Q_3 and Q_4, which represent the basis choice, as a function of Q_1 and Q_2. Now, let us make an assumption that an element of reality of Q_3 (Q_4) will not be a function of Q_2 (Q_1). The probability of obtaining $+1$ and $+1$ for Q_3 and Q_4 when $Q_1=+1$ and $Q_2=+1$ is $1/2$. In the following, let us consider these 50% of the runs where Q_3 and Q_4 will both be $+1$ when $Q_1=+1$ and $Q_2=+1$. According to (3), when $+1$ and $+1$ are obtained from Q_1 and Q_3, we can predict with certainty the outcome of Q_4 will be -1 as long as the outcome of Q_2 is -1. Therefore, according to the reality criterion given by EPR the value for the elements of reality for the outcome of Q_4 as a function of Q_2 is the following,

$$N_{Q_4}(N_{Q_2}=-1)=-1. \tag{4}$$

In the 50% of the runs we are considering, N_{Q_4} in (4) should not change even if Q_1 is -1 instead of $+1$ since we assumed the element of reality for Q_4 will not be a function of Q_1. Similarly, in these runs, when the outcomes of Q_2 and Q_4 are $+1$ and $+1$, respectively, when $Q_1=-1$, Q_3 must yield -1. Therefore, we can deduce the value of the element of reality for Q_3 with respect to Q_1 in these events as

$$N_{Q_3}(N_{Q_1}=-1)=-1. \tag{5}$$

Since we derived the value for the element of reality for Q_3 with no dependence on Q_2, these values should not change when $Q_2=-1$. Along with N_{Q_4}'s independence of Q_1, this contradicts the fact that, according to (3), when $Q_1=-1$ and $Q_2=-1$, the probability of getting -1 and -1 for Q_3 and Q_4 is zero. Therefore, we are led to an inconsistency when we assumed an element of reality for Q_3 (Q_4) cannot be a function of Q_2 (Q_1)'s outcome. If the value for the element of reality for Q_3 can be derived not only as a function of Q_1 but also of Q_2, and similarly for Q_4 as a function of both Q_1 and Q_2, the contradiction can be avoided. While using the logic in [5], we've shown that an element of reality for Q_3 is a function of at least Q_1 and Q_2 while Q_4's reality is also dependent on the same Q_1 and Q_2. Therefore, we conclude that, with respect to the outcome of entangled quantum systems Q_1 and Q_2, Q_3 and Q_4 cannot be independent from each other.

We have shown that if we take the claim in [11], i.e., the observer's reference frame going backwards in time, the assumption in Bell-type proofs, which states that the basis choice at two ends are not correlated at two ends, may not be valid. Certainly, there is always the possibility that two detectors might have interacted a long time ago. However, what we have shown in regards to the correlation of measurement bases is fundamentally different from such possibility. We considered that the basis between X and Z may be chosen by collapsing a superposed qubit. Since this qubit is a pure state, there should not be any correlation with any other system, including a detector, at the other end. Even when the basis was chosen randomly based on quantum probability, we have shown that there is still a possibility that two basis choices are correlated.

Acknowledgements I thank Manny Knill for his valuable suggestions for many improvements on an earlier version of this paper and Gavin Brennen for helpful discussions. I am also grateful to the referees for valuable suggestions.

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